J. Phys. B: At. Mol. Opt. Phys. **36** (2003) 4201–4203 PII: S0953-4075(03)58870-8

COMMENT

Comment on 'Quantum inversion of cold atoms in a microcavity: spatial dependence'

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Received 13 January 2003
Published 9 October 2003
Online at stacks.iop.org/JPhysB/36/4201

Abstract

In a recent work, Abdel-Aty and Obada (2002 *J. Phys. B: At. Mol. Opt. Phys.* **35** 807–13) analysed the quantum inversion of cold atoms in a microcavity, the motion of the atoms being described quantum mechanically. Two-level atoms were assumed to interact with a single mode of the cavity, and the off-resonance case was considered (namely the atomic transition frequency is detuned from the single-mode cavity frequency). We demonstrate in this paper that this case is incorrectly treated by these authors, and we therefore question their conclusions.

The interaction of cold atoms with microwave high-Q cavities (cold atom micromaser, also denominated mazer) has experienced in recent years an increasing interest since it was demonstrated by Scully *et al* [1] that this interaction leads to a new induced emission mechanism inside the cavity. In a recent work, Abdel-Aty and Obada [2] extended the study of the mazer by analysing the quantum inversion of cold atoms interacting with such cavities. In their model, two-level atoms are assumed to move along the z-direction, towards a cavity of length L. The atomic centre-of-mass motion is described quantum mechanically, and the atoms are coupled to a single mode of the quantized field present in the cavity. The usual rotating-wave approximation is made, and, in contrast to previous works, the general off-resonant case is considered (the atomic transition and the cavity mode frequencies are detuned). Their Hamiltonian reads

$$H = \frac{p_z^2}{2M} + \frac{\hbar\Delta}{2}\sigma_z + \hbar\omega\left(a^\dagger a + \frac{1}{2}\sigma_z\right) + \hbar\lambda f(z)(a^\dagger\sigma + a\sigma^\dagger), \tag{1}$$

where p_z is the atomic centre-of-mass momentum along the z-axis, M the atomic mass, ω the cavity field mode frequency, Δ is the detuning, $\omega_0 - \omega$, with ω_0 the atomic transition frequency, $\sigma = |g\rangle\langle e|$ ($|e\rangle$ and $|g\rangle$ are respectively the upper and lower levels of the two-level atom), σ_z is the atomic inversion operator, $|e\rangle\langle e| - |g\rangle\langle g|$, a and $a^\dagger$ are respectively the annihilation and

0953-4075/03/204201+03$30.00

creation operators of the cavity radiation field, λ is the atom–field coupling strength and $f(z)$ is the cavity field mode function.

In their studies, Abdel-Aty and Obada [2] consider an atom initially in the excited state $|e\rangle$, and the field in the arbitrary state $\sum_n D_n|n\rangle$, where $|n\rangle$ is the photon number state. The centre-of-mass motion is described by the initial wavepacket $\int \mathrm{d}k\, G(k)\mathrm{e}^{\mathrm{i}kz}\theta(-z)$, where $\theta(z)$ is the Heaviside step function, indicating that the atoms are coming from the left-hand part to the cavity. For a mesa field mode ($f(z)=1$, for $0<z<L$), they find that the wavefunction, $|\Psi(t)\rangle$, of the atom–field system is (for any time t)

$$\begin{aligned}|\Psi(t)\rangle = \frac{1}{\sqrt{2}}\int \mathrm{d}k\, G(k)\exp\left(-\mathrm{i}\frac{\hbar k^2 t}{2M}\right)\sum_n D_n\\ \times [([\mathrm{e}^{\mathrm{i}kz} + (A_n^+ + A_n^-)\mathrm{e}^{-\mathrm{i}kz}]\theta(-z) + (B_n^+ + B_n^-)\mathrm{e}^{\mathrm{i}k(z-L)}\theta(z-L)\\ + \{\alpha_n^+\mathrm{e}^{\mathrm{i}k_n^+ z} + \beta_n^+\mathrm{e}^{-\mathrm{i}k_n^+ z} + \alpha_n^-\mathrm{e}^{\mathrm{i}k_n^- z} + \beta_n^-\mathrm{e}^{-\mathrm{i}k_n^- z}\}[\theta(z)-\theta(z-L)])|e,n\rangle\\ + (\{\alpha_n^+\mathrm{e}^{\mathrm{i}k_n^+ z} + \beta_n^+\mathrm{e}^{-\mathrm{i}k_n^+ z} - \alpha_n^-\mathrm{e}^{\mathrm{i}k_n^- z} - \beta_n^-\mathrm{e}^{-\mathrm{i}k_n^- z}\}[\theta(z)-\theta(z-L)]\\ + (A_n^+ - A_n^-)\mathrm{e}^{-\mathrm{i}kz}\theta(-z) + (B_n^+ - B_n^-)\mathrm{e}^{\mathrm{i}k(z-L)}\theta(z-L))|g,n+1\rangle],\end{aligned} \tag{2}$$

with the coefficients $A_n^\pm$, $B_n^\pm$, $\alpha_n^\pm$ and $\beta_n^\pm$ given by

$$A_n^\pm = \mathrm{i}\Upsilon_n^\pm \sin(k_n^\pm L)\sin(\theta_n)[\cos(k_n^\pm L) - \mathrm{i}\delta_n^\pm \sin(k_n^\pm L)]^{-1}, \tag{3}$$

$$B_n^\pm = \sin(\theta_n)\mathrm{e}^{-\mathrm{i}kL}[\cos(k_n^\pm L) - \mathrm{i}\delta_n^\pm \sin(k_n^\pm L)]^{-1}, \tag{4}$$

$$\alpha_n^\pm = \frac{1}{2}\left(1 + \frac{k}{k_n^\pm}\right)\mathrm{e}^{-\mathrm{i}k_n^\pm L}\sin(\theta_n)\mathrm{e}^{-\mathrm{i}kL}[\cos(k_n^\pm L) - \mathrm{i}\delta_n^\pm \sin(k_n^\pm L)]^{-1}, \tag{5}$$

$$\beta_n^\pm = \frac{1}{2}\left(1 - \frac{k}{k_n^\pm}\right)\mathrm{e}^{\mathrm{i}k_n^\pm L}\sin(\theta_n)\mathrm{e}^{-\mathrm{i}kL}[\cos(k_n^\pm L) - \mathrm{i}\delta_n^\pm \sin(k_n^\pm L)]^{-1}, \tag{6}$$

and

$$k_n^\pm = \sqrt{k^2 \mp \gamma^2\sqrt{\frac{\Delta^2}{4\lambda^2} + (n+1)}}, \tag{7}$$

$$\Upsilon_n^\pm = \frac{1}{2}\left(\frac{k_n^\pm}{k} - \frac{k}{k_n^\pm}\right), \tag{8}$$

$$\delta_n^\pm = \frac{1}{2}\left(\frac{k_n^\pm}{k} + \frac{k}{k_n^\pm}\right), \tag{9}$$

$$\theta_n = \tan^{-1}\left(\frac{\lambda\sqrt{n+1}}{\sqrt{\frac{\Delta^2}{4} + \lambda^2(n+1)} - \frac{\Delta}{2}}\right), \tag{10}$$

where γ is defined by $(\hbar\gamma)^2/2M = \hbar\lambda$.

As clearly mentioned in [3], if we denote by $|\Phi_n^\pm\rangle$ the atom–field dressed states

$$|\Phi_n^+\rangle = \cos\theta_n|g,n+1\rangle + \sin\theta_n|e,n\rangle, \tag{11}$$

$$|\Phi_n^-\rangle = -\sin\theta_n|g,n+1\rangle + \cos\theta_n|e,n\rangle, \tag{12}$$

Abdel-Aty and Obada obtain equations (2)–(10) of the present comment by considering that the wavefunction components $\Psi_n^\pm(z,t) = \langle z, \Phi_n^\pm|\Psi(t)\rangle$ satisfy the Schrödinger equation,

$$\mathrm{i}\hbar\frac{\partial}{\partial t}\Psi_n^\pm(z,t) = \left(\frac{-\hbar^2}{2M}\frac{\partial^2}{\partial z^2} + V_n^\pm(z)\right)\Psi_n^\pm(z,t), \tag{13}$$

with

$$V_n^\pm(z) = \pm\hbar\sqrt{\frac{\Delta^2}{4} + \lambda^2 f^2(z)(n+1)}. \tag{14}$$

Except at resonance ($\Delta = 0$), this assertion is wrong. Contrary to what these authors claim, the wavefunction components $\Psi_n^{\pm}(z,t)$ do not satisfy equation (13) when a detuning Δ is present. Their conclusions are therefore questioned. Indeed, using the completeness relation

$$1 = \int \mathrm{d}z \sum_n (|z,\Phi_n^+\rangle\langle z,\Phi_n^+| + |z,\Phi_n^-\rangle\langle z,\Phi_n^-|) \tag{15}$$

and projecting the Schrödinger equation

$$\mathrm{i}\hbar \frac{\mathrm{d}}{\mathrm{d}t}|\Psi(t)\rangle = H|\Psi(t)\rangle \tag{16}$$

onto the dressed state basis $\{|z,\Phi_n^{\pm}\rangle\}$ yields

$$\mathrm{i}\hbar \frac{\partial}{\partial t}\Psi_n^{\pm}(z,t) = \int \mathrm{d}z' \sum_{n'} (\langle z,\Phi_n^{\pm}|H|z',\Phi_{n'}^{+}\rangle \Psi_{n'}^{+}(z',t) + \langle z,\Phi_n^{\pm}|H|z',\Phi_{n'}^{-}\rangle \Psi_{n'}^{-}(z',t)), \tag{17}$$

that is (after a straightforward calculation of the matrix elements $\langle z,\Phi_n^{\pm}|H|z',\Phi_{n'}^{\pm}\rangle$)

$$\begin{aligned}\mathrm{i}\hbar \frac{\partial}{\partial t}\Psi_n^{+}(z,t) = {} & \left[-\frac{\hbar^2}{2M}\frac{\partial^2}{\partial z^2} + \left(n+\frac{1}{2}\right)\hbar\omega - \cos 2\theta_n \frac{\hbar\Delta}{2} + \hbar\lambda f(z)\sqrt{n+1}\sin 2\theta_n\right]\Psi_n^{+}(z,t) \\ & + \left[\hbar\lambda f(z)\sqrt{n+1}\cos 2\theta_n + \sin 2\theta_n \frac{\hbar\Delta}{2}\right]\Psi_n^{-}(z,t),\end{aligned} \tag{18}$$

$$\begin{aligned}\mathrm{i}\hbar \frac{\partial}{\partial t}\Psi_n^{-}(z,t) = {} & \left[-\frac{\hbar^2}{2M}\frac{\partial^2}{\partial z^2} + \left(n+\frac{1}{2}\right)\hbar\omega + \cos 2\theta_n \frac{\hbar\Delta}{2} - \hbar\lambda f(z)\sqrt{n+1}\sin 2\theta_n\right]\Psi_n^{-}(z,t) \\ & + \left[\hbar\lambda f(z)\sqrt{n+1}\cos 2\theta_n + \sin 2\theta_n \frac{\hbar\Delta}{2}\right]\Psi_n^{+}(z,t).\end{aligned} \tag{19}$$

We get, for each n, two coupled partial differential equations. In the presence of a detuning, the equations verified by the components $\Psi_n^{\pm}(z,t)$ are much more complicated than at resonance, and the atomic interaction with the cavity can no longer be interpreted as an elementary scattering problem over two potentials $V_n^+(z)$ and $V_n^-(z)$. Equations (18) and (19) only reduce in the interaction picture to the simple form (13) when $\Delta = 0$ ($\theta_n = \pi/4$).

We describe in detail the effects of a detuning on the mazer properties in [4]. We show there that actually no basis exists where equations (18) and (19) would separate over the entire z-axis. Expressions for the probability of finding the atoms in the excited state or in the ground state are explicitly given for the mesa mode function. According to this comment, our results differ significantly from those presented by Abdel-Aty and Obada in [2] and [3].

Acknowledgment

We acknowledge the support of the Belgian Institut Interuniversitaire des Sciences Nucléaires (IISN).

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